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LETTER TO THE EDITOR

Domain walls in random field in two dimensions

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Abstract. Scaling exponents for 2D domain walls are determined exactly via Burger's equations.

Recently it was pointed out (Huse *et al* 1985) that domain walls in the random bond Ising model (RBIM) in 2D are related to the noise-driven Burger's equation. Thus the exponents of the domain wall width and the energy gain can be obtained exactly. Here I show that the domain walls in the random field Ising model (RFIM) in 2D can also be mapped to a Burger's equation of B type (Forester *et al* 1977), while the Burger's equation in Huse *et al* (1985) is of A type.

Following Grinstein and Ma (1983 and references therein) we have the Hamiltonian for the domain walls in 2D RFIM:

$$H=\int \mathrm{d}t\left[\frac{1}{2}\left(\frac{\partial x}{\partial t}\right)^2+\int^x h(x',t)\,\mathrm{d}x'\right] \tag{1}$$

where t describes the transverse direction, x is the domain wall height and h are the random fields. The weight $W(x,t)$ of finding the domain wall passing (x,t) obeys (Huse *et al* 1985, Kardar 1985)

$$\frac{\partial W}{\partial t}=\frac{1}{2}\frac{\partial^2 W}{\partial x^2}+W\int^x h(x',t)\,\mathrm{d}x' \tag{2}$$

and this in turn gives

$$\frac{\partial V}{\partial t}=\frac{1}{2}\frac{\partial^2 V}{\partial x^2}+V\frac{\partial}{\partial x}V+h(x,t) \tag{3}$$

which is Burger's equation in $1+1$ dimensions, where $V(x,t)=\partial(\ln W(x,t))/\partial x$.

Forster *et al* actually described two types of noise-driven Burger's equations (Burger's equations and Navier–Stokes equations being the same in one spatial dimension). They differ only in how the noises are correlated. In momentum space we have

$$\text{A type}\qquad \langle h(k,t)h(k',t')\rangle=2Dk^2\delta(k+k')\delta(t-t') \tag{4a}$$

$$\text{B type}\qquad \langle h(k,t)h(k',t')\rangle=2D\delta(k+k')\delta(t-t'). \tag{4b}$$

I have checked that the calculation (Forster *et al* 1977) for the B-type model is correct. Therefore the exponents for the domain wall in 2D RFIM can easily be deduced:

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the width and energy gain are both linear in the length scale L, i.e. the two exponents are $(1, 1)$, to be compared with $(\frac{2}{3}, \frac{1}{3})$ in RBIM.

The one-loop results are actually *exact* in one spatial dimension for the Burger's equations, whereas higher loops vanish identically (Zhang 1986). This is also why the one-loop results for A-type Burger's equations coincide with the exact results of a fluctuation-dissipation theorem.

The results for RFIM are not new; they can be obtained by the Imry-Ma argument (Imry and Ma 1975) and by a different renormalisation group calculation (Grinstein and Ma 1983 and references therein). My results are valid for all temperatures.

From curiosity, I also examined the case when both random bonds and random fields are present, since both are relevant disorders in $1+1$ dimensions. We have the following recursion relations:

$$d\lambda_1/dl = \lambda_1 - 2\lambda_1^2 - 7\lambda_1\lambda_2 + \lambda_2^2 \tag{5a}$$

$$d\lambda_2/dl = \lambda_2[-3\lambda_1 - 9\lambda_2 + 3] \tag{5b}$$

and the inverse of the width exponent is given by $z = 2 - \lambda_1 - 3\lambda_2$. We see that $\lambda_1 = 0$, $\lambda_2 = \frac{1}{3}$ for the B-type model are no longer fixed points and $\lambda_1 = \frac{1}{2}$, $\lambda_2 = 0$ for the A-type model become unstable fixed points. The stable fixed points $\lambda_1 \sim \frac{1}{14}$, $\lambda_2 = (1 - \lambda_1)/3$ are new but z remains nevertheless the same as that of the B-type model. That is, inclusion of random bonds does not change the scaling behaviour of the domain walls in RFIM, as might be intuitively expected.

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